FormIntegral

`FormIntegral[`*domain*, *form*`]`
is a wrapper for an integral of a *form* over a *domain*.

Details

- `FormIntegral` displays an integral of a differential form without evaluation.
- The *domain* is always displayed as a Tooltip on the text *Domain*, which itself is displayed under the integral sign. This prevents an unwieldy *domain* specification from dominating the display.
- The domain can be specified either by using inequalities joined by Ands or as *Mathematica* Regions. For non-Cartesian coordinate systems the form should include a volume factor. With spherical Regions such as Ball or Sphere *Mathematica* automatically inserts the volume factor so it should be left out of the expression.
- The `FormIntegral` can be evaluated with the `EvaluateFormIntegrals` command. Numerous examples of integrals are illustrated there.

Examples (1)

Basic Examples (1)

In[1]:=
```
<< GrassmannCalculus`
```

Complete examples with evaluations are given on the `EvaluateFormIntegrals` page.

One-dimensional integrals look very much like ordinary integrals.

In[2]:=
```
SetEuclideanNSpace[1, {x}, "Form"]
```

The following sepcifies the integral domain as an inequality.

In[3]:= **FormIntegral**$\left[1 \leq x \leq \sqrt{5}\text{ , } x\,dx\right]$

Out[3]= $\int_{\mathcal{D}omain} dx\, x$

the following uses parameters for the domain specification. This will usually require the use of Assumptions when evaluating the integral. However the Assumptions are only entered at the time of evaluation and do not interfer with the normal unevaluated display.

In[4]:=
```
FormIntegral[a ≤ x ≤ b, x dx]
```

Out[4]= $\int_{\mathcal{D}omain} dx\, x$

The following uses a *Mathematica* Region notation for the domain.

```
In[5]:= FormIntegral[Interval[{1, 2}], x dx]
```

Out[5]= $\int_{\mathcal{Domain}} dx\, x$

The following is a 2-dimensional integral using a 2-form and inequality for the domain.

```
In[6]:= SetEuclideanNSpace[2, {x, y}, "Form"]
```

```
In[7]:= FormIntegral[0 ≤ x ≤ 2 && 0 ≤ y ≤ x, y dx ∧ dy]
```

Out[7]= $\int_{\mathcal{Domain}} y\, dx \wedge dy$

The following is the same integral specified with a Simplex region.

```
In[8]:= FormIntegral[Simplex[{{0, 0}, {2, 0}, {2, 2}}], y dx ∧ dy]
```

Out[8]= $\int_{\mathcal{Domain}} y\, dx \wedge dy$

The following expresses a 2-dimensional integral in polar coordinates using r as the volume factor.

```
In[9]:= SetActiveAssociation[PublicGrassmannAtlas⟦"Polar"⟧];
SwitchBasis["Form"];
```

```
In[10]:= FormIntegral[0 ≤ θ ≤ 2 π && 0 ≤ r ≤ R, r dr ∧ dθ]
```

Out[10]= $\int_{\mathcal{Domain}} r\, dr \wedge d\theta$

But with circular or spherical region specification *Mathematica* automatically inserts the volume factor so we use:

```
In[11]:= FormIntegral[Ball[{0, 0}, R], dr ∧ dθ]
```

Out[11]= $\int_{\mathcal{Domain}} dr \wedge d\theta$

A 3-dimensional integral works analogously.

```
In[12]:= SetEuclideanNSpace[3, {x, y, z}, "Form"]
```

```
In[13]:= FormIntegral[0 ≤ x ≤ 1 && 0 ≤ y ≤ 1 && 0 ≤ z ≤ 1, x y z dx ∧ dy ∧ dz]
```

Out[13]= $\int_{\mathcal{Domain}} x\, y\, z\, dx \wedge dy \wedge dz$

```
In[14]:= FormIntegral[Cuboid[{0, 0, 0}], x y z dx ∧ dy ∧ dz]
```

Out[14]= $\int_{\mathcal{Domain}} x\, y\, z\, dx \wedge dy \wedge dz$

```
In[15]:= SetActiveAssociation[PublicGrassmannAtlas⟦"Spherical"⟧];
SwitchBasis["Form"];
```

```
In[16]:= GrassmannVolumeFactor
CoordinateDomain
```

Out[16]= $r^2\, \mathrm{Sin}[\theta]$

Out[16]= $r > 0\ \&\&\ 0 < \theta < \pi\ \&\&\ -\pi < \varphi \le \pi$

In[17]:= `FormIntegral[0 ≤ r ≤ R && Drop[CoordinateDomain, 1], GrassmannVolumeFactor dr ∧ dθ ∧ dφ]`

Out[17]= $\int_{\mathcal{Domain}} r^2 \text{Sin}[\theta]\, dr \wedge d\theta \wedge d\varphi$

In[18]:= `FormIntegral[Ball[{0, 0, 0}, R], dr ∧ dθ ∧ dφ]`

Out[18]= $\int_{\mathcal{Domain}} dr \wedge d\theta \wedge d\varphi$

See Also

EvaluateFormIntegrals · **PullbackForms** · **BasisPushforward**

Related Guides

- Calculus
- Derivatives
- Grassmann Calculus