# GrassmannComplement

`GrassmannComplement[x]`
places an OverBar over the expression $x$, denoting a Grassmann complement.

## Details

- `GrassmannComplement[x]` and `OverBar[x]` are equivalent input forms for constructing the expression $\overline{x}$. They do not simplify or change the expression in any way.
- Other *GrassmannAlgebra* functions are able to interpret the `OverBar` as indicating that a Grassmann complement operation has been applied to `x`.
- The Grassmann complement is a unary operation, and is `Listable`.
- The Grassmann complement of an expression may be entered by using the $\overline{\blacksquare}$ on the *Basic Operations* palette. Nested Grassmann complements may be entered by continued clicking on the $\overline{\blacksquare}$ button.
- The Grassmann complement is defined only in a metric space and any Grassmann expression in a metric space can be complemented.
- The Grassmann complement is a linear operator.
- The Grassmann complement is used to define the notions of orthogonality and metric. Chapter 5 of the book is devoted to the Grassmann complement.

## Examples (1)

### Basic Examples (1)

In[1]:=
```
<< GrassmannCalculus`
```

These inputs are equivalent:

In[2]:=
```
{GrassmannComplement[x], OverBar[x], x̄}
```

Out[2]= $\{\overline{x}, \overline{x}, \overline{x}\}$

To convert complements of basis elements according to the currently declared metric, you can use `ConvertComplements` For example in a 3-dimensional Euclidean space with basis $\{e_1, e_2, e_3\}$ you would get:

In[3]:=
```
★A; ConvertComplements[{ē1, (e1∧e3)‾}]
```

Out[3]= $\{e_2 \wedge e_3, -e_2\}$

In Grassmann algebra the GrassmannComplement and InteriorProduct have been constructed in such a way that the complement element is orthogonal to the element. So above the $e_2 \wedge e_3$ direction is orthogonal to the $e_1$ direction and the $e_2$ direction is orthogonal to the $\overline{e_1 \wedge e_3}$ direction.

In[4]:= $\{e_2 \wedge e_3 \ominus e_1,\ e_1 \wedge e_3 \ominus e_2\}$ // **ToMetricElements**

Out[4]= {0, 0}

In a 3-space, the Grassmann complement of the Grassmann complement of an element is just the element itself. You can check this by applying `GrassmannSimplify` (or its alias $\star\mathcal{S}$) to $\overline{\overline{\underset{m}{x}}}$.

In[5]:= $\star\mathcal{S}\left[\left\{\overline{\overline{a}},\ \overline{\overline{x}},\ \overline{\overline{\underset{2}{x}}},\ \overline{\overline{\underset{3}{x}}}\right\}\right]$

Out[5]= $\left\{a,\ x,\ \underset{2}{x},\ \underset{3}{x}\right\}$

In general, the Grassmann complement of the Grassmann complement of an element may differ from the element by a sign. For example in a 4-space you will get:

In[6]:= $\star\mathcal{B}_4;\ \star\mathcal{S}\left[\left\{\overline{\overline{a}},\ \overline{\overline{x}},\ \overline{\overline{\underset{2}{x}}},\ \overline{\overline{\underset{3}{x}}},\ \overline{\overline{\underset{4}{x}}}\right\}\right]$

Out[6]= $\left\{a,\ -x,\ \underset{2}{x},\ -\underset{3}{x},\ \underset{4}{x}\right\}$

The complement depends on the current metric. In a 2-space with default metric we get

In[7]:= $\star$**A**; $\star\mathcal{B}_2$; **ConvertComplements**$\left[\left\{\overline{e_1},\ \overline{e_2}\right\}\right]$

Out[7]= $\{e_2,\ -e_1\}$

But with a general (symmetric) metric we get

In[8]:= **DeclareMetric[{{a, b}, {b, c}}]**

In[9]:= **ConvertComplements**$\left[\left\{\overline{e_1},\ \overline{e_2}\right\}\right]$

Out[9]= $\left\{-\frac{b\,e_1}{\sqrt{\star g}}+\frac{a\,e_2}{\sqrt{\star g}},\ -\frac{c\,e_1}{\sqrt{\star g}}+\frac{b\,e_2}{\sqrt{\star g}}\right\}$

The symbol $\star g$ stands for the determinant of the metric tensor.

In[10]:= **ToMetricElements[$\star$g]**

Out[10]= $-b^2 + a\,c$

### See Also

**VectorSpaceComplements** · **VectorSpaceComplement** · **Complement** · **GradeComplement** · **ToVectorSpaceComplements** · **ToGrassmannComplements** · **ConvertComplements** · **Cobasis** · **ComplementPalette**

### Related Guides

- Expressions

- Grassmann Calculus
- Grassmann Complement